

IDENTIFICATION OF ORDINARY DIFFERENTIAL EQUATION SURROGATES FROM NUMERICAL INTEGRATION WITH ADAPTIVE TIME-STEP SCHEMES

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Keywords: recurrent artificial neural networks, ordinary differential equations, adaptive step size Runge-Kutta methods, nonlinear dynamics

Abstract

In various areas of dynamic system analysis like model predictive control or system parameter identification, the determination of future system states based on an actual one is a key task. Classically, this is done by setting up a mathematical model based on physical principles. In case of mechanical systems, the resulting equations of motions are then transformed into 1st-order ordinary differential equations (ODE), which are solved as initial value problems (IVP) by established numerical integration methods to obtain the system state for a specific time interval of interest. This conventional analysis approach, however, requires the availability of a mathematical system description, which can be challenging because of required expert knowledge or even impossible if the physics of the system is not fully understood. In such cases, surrogates based on machine learning may be applied to extract the mapping characteristics of the underlying ODE from data gathered by either performing physical experiments or running black-box simulations. Using a machine learning approach, an optimal mapping from inputs to desired outputs may be found by optimization of adjustable weights of the surrogate. It has been shown that such a surrogate ODE can be set up by training a feed-forward neural network which is embedded into a fixed-step Runge-Kutta method [1]. In the corresponding recurrent neural network resulting from linking those methods, the forward propagation is then done by consecutive numerical integration steps. For fixed-step integration methods, the time-step width directly influences the reachable precision. In case of nonlinear dynamic systems, numerical integration with step-size adaption should be used. As an extension of [1], therefore, the neural network surrogate will be embedded here into an integration scheme with step-size control. For training this neural network, multiple IVPs are solved by forward propagation of multiple initial values up to specified termination times where known state space data serve as training targets. Losses are calculated between each target and corresponding model output to update the weights of the neural network by making use of established backpropagation methods to find an optimal mapping. The feasibility of the proposed approach is

demonstrated by learning the surrogate ODE of an example multibody system embedded into the widely used RK54(7)M scheme introduced by Dormand & Prince [2]. Therefore, the RK54(7)M algorithm is slightly modified for numerical stabilization of various function derivatives to be connected to the superior model backpropagation graph. Furthermore, batch processing techniques are developed for forward propagation of multiple IVPs in parallel, each with its individual adaptive step-size control

References

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