

A ROTATION-BASED REGULARIZATION FOR THIRD MEDIUM CONTACT WITH APPLICATIONS TO GRADIENT-BASED OPTIMIZATION

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Abstract

Contact modeling remains a major challenge in structural mechanics and computational design optimization. Classical contact formulations rely on explicit surface detection and projection algorithms, which introduce non-smoothness, algorithmic complexity, and difficulties when incorporated into gradient-based optimization frameworks. In contrast, the third medium contact (TMC) method models contact implicitly by embedding interacting bodies in a fictitious compliant medium. Contact constraints are then enforced through constitutive response rather than geometric search, which renders the formulation naturally differentiable and highly attractive for structural and topology optimization.

A fundamental challenge in TMC is the severe distortion of the fictitious medium under compression. Without stabilization, elements may collapse or exhibit excessive warping, leading to loss of robustness and poor numerical performance. Existing regularization strategies either penalize higher-order deformation measures which requires second-order elements, or introduce auxiliary fields to approximate rotation gradients, significantly increasing the number of degrees of freedom. Both approaches limit efficiency and scalability in large-scale optimization problems. In this work, we propose a new regularization strategy based on a geometric measure of local rotational variation. Rather than penalizing the full deformation gradient or its higher-order derivatives, we target only changes in rotation. The key observation is that rotations belong to $SO(3)$, where distances must be measured using an appropriate metric. We employ a rotation metric based on the Frobenius norm of the relative rotation, which provides a consistent and computationally convenient measure of rotational deviation. The regularization is formulated as a penalty on the deviation between the current rotation and a local reference rotation from the previous load step. This yields a formulation that mimics a rotation-gradient penalization while avoiding higher-order derivatives and auxiliary fields.

The resulting formulation exhibits several properties that are particularly beneficial in optimization contexts:

- (i) it exclusively penalizes rotational modes and avoids artificial stiffening of stretch deformation modes,
- (ii) it is directly compatible with first-order finite elements and unstructured meshes,
- (iii) it preserves differentiability of the global energy functional, and
- (iv) it introduces only a modest computational overhead compared to the unregularized TMC formulation.

Benchmark problems in plane strain and fully three-dimensional large-deformation settings demonstrate robust contact behavior and excellent agreement with classical penalty-based contact formulations. At the same time, the proposed regularization significantly reduces computational cost compared to Hessian-based or multi-field stabilization techniques.

By grounding the regularization in a consistent metric on the rotation group, the present work provides a geometrically motivated and computationally efficient stabilization of third medium contact. The formulation is particularly well suited for large-scale gradient-based structural optimization, where robustness, differentiability, and low computational overhead are essential.