

SIMPLE PARAMETRIZATIONS FOR EXTREMAL ELASTIC MICROSTRUCTURES

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Abstract

Additive manufacturing enables the fabrication of complex mechanical components with highly tailored internal architectures. To reduce print time and achieve lightweight parts with a prescribed external shape, the printed structures are typically constructed with a porous infill structure. This creates an opportunity to tune stiffness, strength, weight, and energy absorption by controlling the geometry of the material inside the part. Optimal infill architectures are therefore essential for applications ranging from lightweight aerospace components to high-performance sporting equipment. Achieving high stiffness at low relative density while avoiding stress concentrations remains a key challenge, and analytic parameterizations of optimal micro-structures are valuable both for design and for integration into multi-scale optimization frameworks.

Extremal single-inclusion micro-structures, commonly referred to as Vigdergauz structures, are known to attain the theoretical maximum stiffness predicted by the Hashin–Shtrikman bounds while simultaneously minimizing stress concentrations. Traditionally, their geometries have been represented either through multi-term series expansions [1] or through bitmap-based parameterizations obtained via topology optimization [2]. This work introduces a compact and fully analytic description of Vigdergauz-like extremal micro-structures constructed using smooth-maximum unions of simple signed distance functions. For two-dimensional configurations with elastic square symmetry or isotropy, the near-optimal geometry can be defined using only two parameters. For three-dimensional micro-structures, two parameters also suffice to describe closed-walled and open-walled elastic cubic-symmetric architectures, although only the closed-walled architectures remain extremal.

The global and highly compact nature of these parameterizations offers several advantages. They enable efficient modeling without the need for high-resolution grids, facilitate smooth control of graded relative densities, and integrate naturally with gradient-based optimization frameworks, including homogenization-based topology optimization. Overall, the proposed parameterizations provide a simple yet powerful means of representing extremal micro-structures, opening new

possibilities for design, optimization, and fabrication of high-performance additively manufactured components

References

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