

SHAPE OPTIMIZATION OF NONLINEAR RESONANT RESPONSE IN 3D MEMS STRUCTURES USING SECOND-ORDER NORMAL FORM THEORY

Marian Hörsting, Tobias Marx, Matthias Wenzel, Simon Pflingstl, Peter Degenfeld-Schonburg

Robert Bosch GmbH - Corporate Research

Keywords: Nonlinear Structural Optimization, MEMS/NEMS Resonator Design, Gradient-Based Shape Optimization, Finite Element Modeling, Nonlinear Dynamics Control

Abstract

High quality factor nano- and microelectromechanical systems (NEMS/MEMS) resonators frequently exhibit pronounced geometric nonlinearity, manifesting as amplitude-dependent frequency shifts. Depending on the application, this nonlinear behavior can be strategically exploited in energy harvesters, frequency combs, or resonant accelerometers, or must be suppressed to preserve performance in precision instruments such as gyroscopes and micromirrors. Designing structures with prescribed nonlinear characteristics remains a fundamental challenge, as the relationship between geometry and nonlinear response is highly non-intuitive. Existing optimization approaches are often restricted to simple beam structures due to costly sensitivity calculations and limiting modeling assumptions, such as static condensation, which neglects inertial nonlinearity and fails for structures like cantilevers.

This work presents a gradient-based node-based shape optimization method for controlling the nonlinear amplitude-frequency relationship of arbitrarily complex 3D structures discretized with higher-order finite elements. The nonlinear response is characterized by the effective Duffing coefficient y_{eff} , derived from second-order direct normal form theory. Unlike simplified models that consider only the cubic four-wave coupling term, our formulation accounts for the contributions of all slave modes through nonlinear normal mode theory, capturing both geometric and inertial nonlinearity. A Neumann series approximation of the normal form correction vectors avoids inversions involving shifted stiffness matrices, yielding a computationally efficient evaluation that requires only standard stiffness and mass matrix operations and a small number of linear system solves.

The complete analytical sensitivity of y_{eff} with respect to nodal shape parameters is derived, encompassing eigenvector and eigenvalue dependencies through an adjoint formulation, as well as direct parameter sensitivities of the nonlinear coupling coefficients, stiffness, and mass matrices. This enables the use of the Method of Moving Asymptotes (MMA) for solving the constrained nonlinear

optimization problem with rapid convergence. Manufacturing constraints on minimum structural thickness and gap widths are enforced directly, and eigen frequency constraints preserve operational characteristics throughout optimization. The method is validated against published experimental data for clamped-clamped silicon microbeams, showing improved agreement compared to prior numerical predictions. Three test cases of increasing complexity demonstrate the capabilities. For clamped-clamped beams, optimizations with and without frequency constraints achieve up to 61% reduction and 379% increase in y_{eff} , closely matching literature benchmarks. For a cantilever beam, where inertial nonlinearity is significant and static condensation methods are inapplicable, the method successfully transitions the response from hardening to softening (118% reduction), enhances hardening (533% increase), and achieves complete linearization ($y_{\text{eff}} \approx 0$). For a MEMS gyroscope of industrial complexity with constrained drive and sense mode frequencies, the optimizer achieves $\pm 1000\%$ changes in y_{eff} as well as full nonlinearity elimination through subtle geometric modifications that would be impossible to identify by manual design iteration.

The proposed method provides an accurate, efficient, and general-purpose framework for tailoring nonlinear resonant behavior in MEMS design, enabling precise control from strong hardening through linear to strong softening responses for arbitrary 3D geometries under realistic manufacturing and operational constraints.