

A NUMERICAL FRAMEWORK FOR FORM FINDING OF GRIDSHELLS CONSIDERING FABRICATION UNCERTAINTIES

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Abstract

The Force Density Method (FDM)[1] enables the form-finding of reticulated shells by exploiting the equilibrium conditions of bar structures. By performing a change of variables and writing the static equilibrium equations in terms of the ratios of force to length in each bar -the so-called force densities - the resulting system becomes linear and uncoupled in the nodal coordinates. By controlling these force densities, it is possible to generate shapes capable of sustaining the applied loads. For a statically indeterminate network, however, the set of forces associated with prescribed force densities represents only a statically admissible solution, and a finite element analysis is generally required to recover the displacements by solving the elastic equilibrium. This can be done efficiently, since the form of the global stiffness matrix expressed in terms of the force densities governing the shape of the network can be easily derived and computed [2]. A FDM-based method of Robust Design Optimization is introduced in this contribution, tailored to elastic gridshells affected by fabrication uncertainties [3]. Building on an established perturbation approach [4], an augmented deterministic form-finding problem is formulated. This avoids the computational costs associated with probabilistic formulations and Monte Carlo-based optimization algorithms, relying on a single factorization of the FDM-based stiffness matrix at each iteration while solving the same elastic problems for multiple load cases.

To minimize the influence of stochastic variability, the set of force densities corresponding to a spatial geometry that minimizes a weighted sum of the mean compliance and its standard deviation is sought using sequential convex programming. Exploiting the formalism of FDM to govern the dependence of both the shape and the global stiffness matrix on the varying network geometry is particularly advantageous for computing sensitivities. Numerical simulations address reticulated shells subjected to gravity loads and a lightweight column-like element related to metal additive manufacturing, investigating the sensitivity of the

optimal shape to fabrication uncertainties affecting either the elastic modulus or the area of the bar cross-sections. Numerical features of the proposed approach and the sensitivity of the obtained results are discussed. Extensions to other types of uncertainties are foreseen

References

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