

DISTRIBUTIONALLY ROBUST SHAPE AND TOPOLOGY OPTIMIZATION

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Abstract

Shape and topology optimization generally aims to minimize a cost function of the domain, possibly under constraints. In applications, this cost function involves the solution to a boundary-value problem encoding its physical behavior; the latter typically involves data, such as the applied forces or the properties of the constituent material in the realm of mechanical structures, the viscosity in fluid mechanics, etc. Most often, these physical parameters are known imperfectly, be it because they are measured with error-prone processes, or because they are altered with time. This realization raises the need to incorporate a degree of awareness about such uncertainties into the optimal design problem, and two different paradigms are commonly used in the literature to achieve this purpose: Worst-case approaches are suitable when no information is available about the uncertain parameters, except for an upper bound on their amplitude around a given mean value. One then minimizes the worst (i.e. largest) value of the cost functional over the range of possible scenarios. Beyond the mathematical complexity of the resulting bilevel min-max problem, this approach often yields designs with poor nominal performance, for the sake of anticipating an unlikely worst-case event. Probabilistic approaches describe the uncertain parameters as random variables and rely on the knowledge of their law. One then typically minimizes the mean value, or the standard deviation of the cost function under the possible uncertainties, e.g. by Monte Carlo sampling strategies, stochastic Galerkin resolutions of the boundary value problem at play... This approach in turn suffers from a major conceptual drawback: in practical situations, the law of the parameter uncertainty is unknown -- at best can it be reconstructed from a few observed samples.

This presentation aims to overcome the latter flaw of probabilistic robust optimal design methods by introducing the paradigm of distributionally robust optimization in this context. This recent viewpoint in convex optimization acknowledges that in realistic situations, the law of the uncertain parameters is itself unknown, and it consists in minimizing the worst possible value of the expected cost, when the law

of the uncertain parameters lies within a certain "distance" from this nominal law. Recent findings in optimal transport theory and convex optimization make it possible to reformulate the a priori intricate, resulting bilevel min-max problem into a single level minimization problem which can be tackled by "classical" optimal design algorithms.